

h astrometric. How to make protractor (sextant), that has step of 2 seconds, and measurements go directly to computer?

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A number of books and articles are devoted to the navigation of the sky (including the Sun as reference) and the measuring devices for it [1]. The basis of all of them is a circle and the drawing of the angle formed in it, measured in degrees. This is followed by calculation of the trigonometric functions. As it is known, they can only be computed by expanding them into infinite series of functions. Many different graphical methods are used for these calculations everywhere. Much mathematical work has been devoted to celestial navigation [2]. Everything is done on the basis of spherical trigonometry. Simpler mathematical models are used for practical calculations [3]. Let us take one example that is presented in the paper [3]. This is a sine altitude calculation. The mathematical model of the sine altitude is:

$$\sin(\text{alt}) = \sin(\text{lat}) \cdot \sin(\text{dec}) + \cos(\text{lat}) \cdot \cos(\text{dec}) \cdot \cos(\text{LHA}) \quad (1)$$

Sine and Cosine are calculated using the infinite series method. To compute (1) two sine and three cosine lines requires some computer processing time. In (1) alt - altitude, lat - latitude, dec - declination, LHA - Local hour angle. Let us compute just one size $\sin(\text{lat})$. In paper [3] given latitude is $-38^{\circ} \times 34,2^{\text{I}}$. At the beginning it is needed to convert the minutes into parts of a degree. Then convert the degrees to radians we will get:

$$A=0,6731 \quad (2)$$

Now we can count using an infinite line.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

We end up getting

$$\sin(\text{lat}) = 0,62347 \quad (3)$$

It is what is given in work [3].

Let us use h geometry methods.

Let us recall that in h geometry, the sine (sph) is computed:

$$\text{sph} = \frac{h}{\sqrt{h^2 + (1-h)^2}} \quad (4)$$

In order to compare the sine calculations by the classical series method (3) with the h geometry sph (4), the angle given in radians (2), they will be converted to the magnitude of that angle when it is measured by the parameters h. The conversion formula is:

$$h = \frac{\tan(\alpha)}{1 + \tan(\alpha)} \quad (5)$$

Instead of the angle size measured in radians (2) we get the angle size measured in h parameters.

$$h = 0,44365 \quad (6)$$

Using (4) we get

$$\sin(\text{lat}) = 0,62347 \quad (7)$$

As we can see, the calculation results (3) and (7) completely coincide. Only when calculating h by the geometry method, the computer computation time is significantly shortened. Here, we computed only one sine. Let us write the expression of sin altitude calculation (1) not with classical trigonometric functions, but with h geometry functions. Recall that:

$$\text{cph} = \frac{1 - h}{\sqrt{h^2 + (1 - h)^2}} \quad (8)$$

In the altitude expression (1), let us change sin and cos to sph (4) and cph (8). Let's introduce the formulas:

$$H = \sqrt{h^2 + (1 - h)^2} \quad (9)$$

Answer we get from (1)

$$\sin(\text{alt}) = \frac{1}{H_{\text{lat}} \cdot H_{\text{dec}}} \cdot \left[h_{\text{lat}} \cdot h_{\text{dec}} + (1 - h_{\text{lat}}) \cdot (1 - h_{\text{dec}}) \cdot (1 - h_{\text{LHA}}) \cdot \frac{1}{H_{\text{LHA}}} \right] \quad (10)$$

Let us compute sin (alt) (1), where classical trigonometric models are used, and compute sin (alt), using mathematical models of h geometry. The calculation results are absolutely the same. Understandably, calculating sin (alt) with model (10) is much simpler.

How to measure angles when you need to calculate sin (alt) (1) or (10).

The problem of measuring the size of an angle arose as soon as trigonometry was proposed. It is a long and interesting story described in books and articles. One of the oldest and most widely described angle measuring devices is the sextant. It turns out sextants are produced and now days and their use has come out of use only in ships. They are used not only in airplanes, but even in space. After all, planets cannot be "extinguished", and information from artificial Earth satellites can be falsified, or the satellite can simply be damaged by opponents. The work of the sextant is based on the fact that a metal is made in a circle, or part of it, and the whole circle is divided into 360 parts like a circular ruler. Each part represents one degree. The space between the two dashes denoting the degree of degree is divided into 60 parts, each denoting one minute of the grave. Such metal circles are also made for telescopes, where the diameter of such a circle can be 3 or 6 meters. The larger the diameter of such a circle, the more places are marked by 60 lines denoting the minutes of the angle. This is still the case with telescopes. It is understood that in the production of a sextant with a circle diameter not exceeding one and a half meters, putting together all 60 braces, denoting the minutes of the angle, problems arise in precision mechanics. Dashes to measure in seconds are no longer discussed.

h astrometry

Let us start with the classic where the angle is placed in a circle.

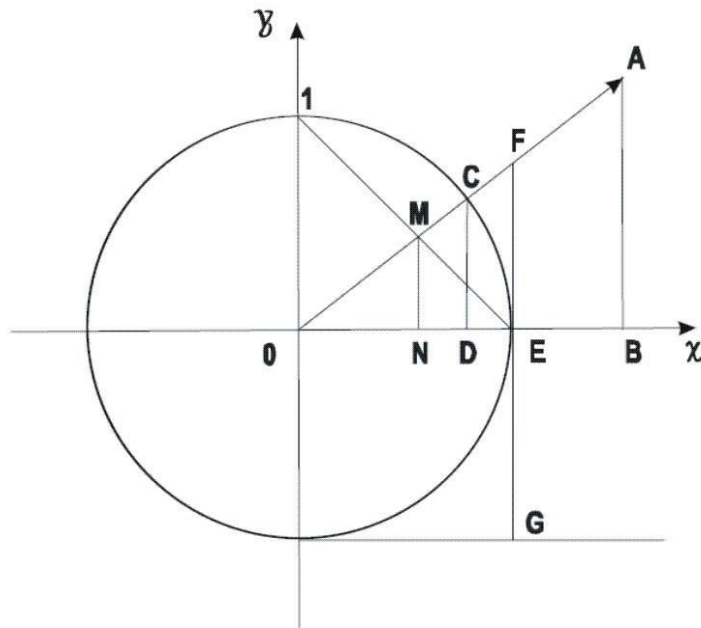
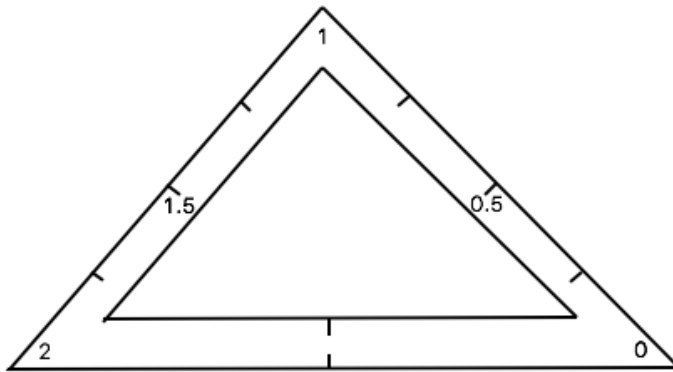


Fig. 1-2

Fig.1-2 shows the circle in which the inserted angle $COE = \alpha$ is measured in degrees (radians). The angle COE is measured by the arc length CE . How to measure the arc length of the CE turns out to be a problem. Sextants and even telescopes make a metal arc (part of a circle, or the whole circle) on which dashes and numbers representing the magnitude of an angle are measured in degrees. The number written there must be scanned by a person and entered in a computer.

We in our proposed h geometry, we proposed to measure the magnitude of the angle COE in the straight line MN . The length of the straight section MN is denoted by the letter h . It is possible to write $MN = h$. You can read more about this [4]. It was first proposed in 1987. [5] At the conference of mathematicians held at Vilnius University, which was also organized by the Institute of Mathematics and Cybernetics of the Lithuanian Academy of Sciences, where I (Donaldas Zanevicius) read the report "Where the Parabolic Sine Has Disappeared". The most famous Lithuanian mathematicians of that time (academics, professors) who took part here listened to my message and kept silent as if they had just drunk a sip of water. That was the end of it. For the first time, an official word of praise for the geometry of h was expressed by a Ukrainian academician at a scientific conference in Kiev. Much later (in 2011), I was invited to come to India and make a presentation at the First International Science Congress there. This was followed by the publication of the ISC-2011 Report Book [5]. After we published the article in a magazine published in (VGTU) in Lithuania that was called: "GEODESY AND CARTOGRAPHY" 201 36(4); 160-163, kur h geometrijos idėjos naudojamos geodezinių uždavinių sprendimui jį išsivertė į anglų kalbą ir pacitavo Harvard's Department of Astronomy, Smithsonian Astrophysical Observatory and NASA Astrophysics Data System [6]."

Well, now let's get back to the drawing. (Fig.1-2) **let us forget the circle**. From the triangle EO , and in the second quadrant of the same triangle, let us make a triangle with a ruler (Analogous to the protractor now used in schools).



Like a protractor, an angle gauge can be used. After placing it on the drawing drawn on paper, or wooden or metal structures. It is not difficult to make such a triangular ruler yourself. With it we will measure the magnitude of the angle with the parameters h . But again, it takes a person to scan what is the h angle drawn. The right-angle h is equal to one. Let us take a look back at the drawing again. (Fig.1-2) Let us form a triangle $EO1$ and an allow OM rotating about a point O . Instead of $E1$, let us insert a linear electronic ruler whose slider (at point M) is moved by the arrow OM . The electronic ruler at point E indicates 0 (zero) and at point 1 indicates (one). The sliding slider OM , at point M , indicates the value of h , which can vary from zero to one. The h value displayed by the slider immediately transmits the h value to the computer. We will call all this the h meter. Such a h meter was manufactured by UAB Baltgina (Panevėžys), whose director is Faustas Keršys. Lukas Keršys also worked with F.Keršys. Lukas Keršys is currently studying in the USA. (We call such a meter the h -meter DFL.) The first such h -meter was manufactured in 2020, May. His photo is shown. In the next photo you can see that the h meter shows.

$$h = 0.82482 \quad (11)$$

What does this mean? This means that the h meter measures the h parameter to five decimal places. And what does it mean if h is converted to degrees, minutes and seconds? We can use dependency.

$$\alpha_h = \text{atan}\left(\frac{h}{1-h}\right) \quad (12)$$

We will get. $\alpha_h = 1.361520 \quad (13)$

And converted $\alpha_{ho} = \frac{180}{\pi} \cdot \alpha_h \quad (14)$

We will get $\alpha_{ho} = 78.0090 \quad (15)$

Calculate what it means in seconds if

$$h_{oo} = 0.000010 \quad (16)$$

Using (12) we will get (radians)

$$\alpha_{hoo} = 10^{-5} \quad (17)$$

Assuming that one degree has 3600 seconds, we get (16) h_{oo} is

$$2.063 \text{ seconds} \quad (18)$$

Means h meter measures with an accuracy of 2.063 seconds.

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Literatūra.

1. Viki Moore Ocean Yachtmaster. [Ocean Yachtmaster | Astrolabe Sailing](#)
2. Richard LAO Mathematics for Celestial Navigation. USA 2018
https://www.researchgate.net/publication/330842260_Mathematics_for_Celestial_Navigation
3. Nautical Almanac. Celestial Navigation.
[Celestial navigation : formulas and calculations \(nauticalalmanac.it\)](#)
4. Dr.Donaldas Zanevičius Paradigma matematikoje. Matematika be π .
[Paradigm | Kosmose.lt](#)
5. Donaldas Zanevičius Mathematics without π and $\sin\alpha$, $\cos\alpha$ (when angle α is being measured in degrees).
11th International Science Congress (ISC-2011) India 24-25 Dec 2011 p.193
PDF|ISC-2011 - ISCA : International E-Publication
www.isca.co.in/E-Souvinor%202011.pdf
6. Dr.Donaldas Zanevičius, Faustas Keršys TECHNOLOGIES FOR CALCULATING GEODETIC COORDINATES APPLYING H-GEOMETRY FUNCTIONS. (The article was written in Lithuanian).
GEODESY AND CARTOGRAPHY 201 36(4); 160-163 This article was cited in various international publications, including Harvard's Department of Astronomy, Smithsonian Astrophysical Observatory and NASA Astrophysics Data System
<http://adsabs.harvard.edu/abs/2010GeCar..36..160Z>